Ship Squat; An Analysis of Two Approximation Formulas Using the Physics of Hydrodynamic Flow

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Abstract

Ship squat, the downward pull of vessels traveling at speed in restricted waters, is of great practical importance given the possibility for grounding, the implications for dredging, and the potential for sinkage and even loss of life. Since the groundbreaking work of Tuck, many formulas have surfaced which attempt to calculate and predict the actual amount of downward displacement given the size, shape, draught, under-keel clearance, and, most importantly, speed of the moving vessel. The formulas are, however, of limited utility because of their mathematical complexity; mariners often want a rough but robust estimate of predicted downward displacement without having to resort to a computer and spreadsheet. We analyze two such approximation formulae here, one due to Barrass and the other due to Schmiechen, and argue that one is correct based on physical principles at high speeds. The other must lead to false results at significant speed where squat is appreciable. We argue that the correct dependency of squat on speed must be velocity cubed (V^3), versus velocity squared (V^2), in the critical limit where the Froude depth number approaches unity.

1. Introduction

Since the groundbreaking theoretical work of Tuck (Tuck, 1967), many theoretical as well as empirical formulas have surfaced (PIANC/IAPH WG 30D, 1994; Vantorre, 1995; Landsburg, 1995, 1996) which attempt to mathematically predict the amount of downward displacement (squat) of a vessel given the ships' profile, physical dimensions, and speed in restricted waters. These formulas tend to be complicated in that a computer and spreadsheet are often needed to evaluate, i.e., calculate the actual amount of squat. Furthermore, even with a spreadsheet, a significant bandwidth, or spread, in the predicted squat amount is obtained from the different formulae given the

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same initial input parameters. (Qualitatively, however, the results look rather similar.) The problem rests with the fact that many parameters are involved, and it is difficult to correctly ascertain them empirically at any one instant in time in other than controlled conditions. Secondly, we are dealing with a problem in hydrodynamic flow approaching critical phenomena where a small variation of one parameter can lead to a remarkably different and dramatic change in the outcome as a whole.

Even if we decide on a formula giving middle-of-the-road predictions for squat such as the Huuska/Guliev formula adopted by ICORELS (Huuska, 1976; PIANC/IAPH WG 30D, 1994; Landsburg 1995, 1996) and which is a direct extension of Tucks' original work, the problem still remains to define a useful approximation formula. Barrass (Barrass, 1979a; Barrass, 1979b; Barrass, 1981; PIANC/IAPH WG 30D, 1994) has come up with a popular simplified formula (his second iteration) and we show in this paper that his latest and improved formula (Barrass II) can be derived from the more general Huuska/ICORELS formulation. As far as we know, Barrass' formula has never been derived from a more extensive underlying theory. His formula was based on empirical work having studied and analyzed about 300 cases of squat involving actual ships and ship-models. The motivation of Barrass for finding a simplified formula was simple. He was motivated by the desire to come up with a formula which could be utilized on ships by ship pilots which was accurate and which didn't rely on trial-and-error, rule-of-thumb guesstimates.

In this paper we will analyze two such approximation formulas. One is due to Barrass and the other originates with Schmiechen (Schmiechen, 1997). Both can be derived from the more general Huuska/ICORELS formulation as we shall show here. However, they each give remarkably disparate results. Our objective is to show why this is so and why one approximation is correct and the other limit is physically incorrect when there is significant squat. The key is to recognize and understand the correct limiting process at the outset of turbulence. Even though both results are physically correct, the physics will show that *only* one result is valid in the realm of practical interest.

In section 2 we derive what is essentially the Barrass II formula by taking a certain limit of the Huuska/ICORELS equation and making certain realistic assumptions. In section 3 we take a different limit of the Huuska/ICORELS equation and obtain Schmiechens' formula. Results are then compared from a physical perspective. Our conclusions and recommendations are presented in section 4, our final section.

2. BARRASS' FORMULA AS AN APPRIMATION OF HUUSKA/ICORELS

Ship squat is the tendency of a vessel to sink and trim when underway thereby reducing its' under-keel clearance. The downward pull can be measured as a displacement, S_M, in meters (m). The trim is typically measured as a rotation, , in radians (rads) about the horizontal transverse axis of the ship, i.e., about a line going through the beam. The total squat at the bow is given by the equation

$$S = S_M + ½ L_{pp'} \tag{1}$$

where $L_{pp'}$ is the length between perpendiculars of the at-rest vessel by the waterline.

Squat is a hydrodynamic effect which depends critically on the speed of the vessel, but also on the draught, the length of the ship, the shape of the hull, and the under-keel clearance. A very good formula giving solid middle-of-the-road estimates for the actual squat displacement is the Huuska/ICORELS formula mentioned above. For simplicity we assume an open waterway laterally, i.e., no breath restrictions in a horizontal sense. Then that formula simplifies to:

$$S = 2.4 \cdot \Delta / (L_{pp'})^2 \cdot F_{nh}^{\ 2} / \sqrt{(1 - F_{nh}^{\ 2})} \tag{2}$$

In equation (2), S is the same displacement as in equation (1), measured in meters and is due to a ships' motion in shallow water, and Δ is the underwater volume of the vessel in cubic meters. Δ can be calculated as $\Delta = CB \cdot Lpp' \cdot B \cdot T$, where Lpp' is the length of the ship between perpendiculars (in m), B is the maximum beam width of the ship by the waterline (in m), and T is the ships' draught, or depth in the water, when at rest (again in m). The block coefficient, CB, is a unit-less ratio which measures the actual submerged volume of the ship in relation to a corresponding submerged rectangular block volume. By definition, CB = (submerged volume)/($Lpp' \cdot B \cdot T$). The Froude depth number, Fnh , is another dimensionless, i.e. unit-less, scale parameter defined as $V/\sqrt{(g \cdot h)}$ where V is the speed of the vessel in m/s, h is the undisturbed water depth in meters (m) and g is the acceleration due to gravity, 9.81 m/s2.

The block coefficient, CB, measures how streamline the hull of a vessel is. For bulky tankers, CB is about .85, whereas for finer formed vessels, CB is approximately .6. Maximum squat typically occurs at the bow (front) of the ship but for high speed vessels with a block coefficient less than about .7, the squat can actually occur at the stern (rear) of the vessel. We keep in mind that Δ is measured in cubic meters where a one cubic meter displacement means one metric ton (1000kg) of fresh water has been pushed aside. In seawater, a one cubic meter displacement means even more water mass has been pushed aside, approximately 1030 kg, due to the higher density of seawater. By Archimedes principle, the upward buoyant force acting on a vessel keeping it afloat is equal to the weight of the fluid which has been displaced, i.e., pushed aside. Obviously, by equation (2), we see that the amount of expected squat is directly proportional to Δ. Δ, in turn, is directly proportional to T, the draught in the water.

Bearing this in mind, equation (2) can be recast in dimensionless form; we can rewrite it as

$$S/T = C2 \cdot Fnh2/\sqrt{(}1 - Fnh2) \qquad (3)$$

where C2 is a constant, defined by C2 = 2.4 · CB · B /Lpp' ,which depends only on the specific dimensions of the ship. The unit-less ratio, S/T, is the amount of squat in relation to the original at-rest draught of the vessel. From equation (3) it is clear that the Froude depth number, Fnh, is what we should focus on for hydrodynamic purposes since it is this term which depends on the speed of the vessel.

When a ship moves through water it creates a captive wave much like an aircraft when moving through the air. An aircraft can move at subsonic speeds where (Vobj/ Vsnd) < 1, at critical speed where (Vobj/ Vsnd) = 1, or at supersonic speeds where (Vobj/ Vsnd) > 1. Vobj stands for the speed of the aircraft, and Vsnd refers to the speed of sound. Aircraft approaching the critical speed, Vobj   Vsnd (Mach 1), require much greater horsepower (in fact, an exponential increase is necessary) in order to overcome the increased aerodynamic resistance. This increased aerodynamic resistance is caused by the accompanying captive wave of the aircraft which it is now being overtaken. If the aircraft has sufficient horsepower, then it can literally punch its way through this envelope, break free of its captive wave, and, as a consequence, a sonic shock wave is produced. This can only happen when it goes at critical speed, or supercritical speeds.

So too with ships but now the resistance is due to a ship attempting to push its way through an envelope created by water which is, of course, a hydrodynamic (versus aerodynamic) effect. When attempting to overtake its captive water wave, a vessel requires additional and significant horsepower. For most large ships with substantial underwater draughts, this is virtually impossible... the hp is not sufficient. With certain fast moving ships, however, it is possible (even with significant draught) to go "critical" and "supercritical" if the horsepower is there. Then the vessel would literally lift up and out of the water in order to travel at these high speeds. We are thinking of speedboats, hydrofoils and hovercraft, which can travel at high speeds but only after lifting up and out of the water. Horsepower is what it takes to break free from the captive wave.

The Froude depth number is the hydrodynamic equivalent of (Vobj/ Vsnd). As Fnh approaches one, the hydrodynamic resistance to motion increases exponentially. Fnh < 1 denotes subcritical speeds, Fnh = 1 defines critical speed, and Fnh > 1 indicates supercritical conditions which can be achieved by speedboats, hydrofoils, etc. Typical upper values for Fnh are Fnh   .6 for tankers and Fnh   .7 for container ships, ships which both have sizable draughts. These vessels cannot simply lift up and out of the water for the required hp would be astronomical.

In a sense, Fnh = 1 determines the effective speed limit for vessels of sizable draught. In fact this speed limit depends on the depth of the water because $Fnh = V/\surd(g{\cdot}h)$, and *h, in turn, measures the water depth in meters.*

It is well-known experimental fact (PIANC/IAPH WG 30D, 1994) that ship squat is not significant for V < 6 knots = 3.08 m/s (1 knot = 1.15 mph = .514 m/s), i.e., for Fnh < .3. This has been shown empirically under a variety of conditions. Yet it is precisely in this limit that equation (2) reduces to the Barrass II formula. To see this we assume *that Fnh is small, i.e., significantly less than one.* Using the binomial expansion

$$Fnh2/\surd(1- Fnh2) = Fnh2 \cdot (1 + ½\, Fnh2 + \ldots) \qquad (4)$$

Since Fnh2 << 1, we keep only the first term on the right hand side of the expan-*sion.* Thus, by equation (2)

$$S \quad 2.4 \cdot CB \cdot B \cdot T \cdot Fnh2/ Lpp' \qquad (5)$$

$$(2.4 \cdot CB \cdot B/ Lpp') \cdot V2/ (g \cdot (h/T))$$

$$(.245 \cdot CB \cdot B/ Lpp') \cdot V2/ (h/T)$$

This can be further simplified. For the vessels that we are considering, the length to beam ratio is typically about 7. Some examples will substantiate this claim. For 250,000 tdw tankers (280,500 tonnes loaded), typical dimensions are Lpp'/B = 330m/50m = 6.6. For 65 tdw bulk carriers (85,000 tonnes loaded), typical values are Lpp'/B = 245m/35m = 7. And for Panamax container ships (65,000 tonnes loaded), good representative values are Lpp'/B = 270m/32m = 8.44. Thus, with the approximation that Lpp'/B   7, equation (5) can further be reduced to

$$S \quad .035 \cdot CB \cdot V2/ (h/T) \qquad (6)$$

It is this expression which we wish to compare to Barrass' latest formula (Barrass II).

The Barrass II formula, obtained as a result of analyzing about 300 actual squat results, some measured on ships and some measured on ship-models, reads

$$S \quad (CB/ 30) \cdot (S2)\ 2/3 \cdot Vkt2.08 \qquad (7)$$

In equation (7), Vkt is the speed of the vessel in knots and S2 is the blockage ratio (sometimes referred to as the velocity return factor) defined as

$$S2 = As/ Aw = As/(Ac - As) = S1/(1- S1) \tag{8}$$

In (8), As is the submerged mid-ship cross-sectional area, and Ac is the cross-sectional area of the channel including that of the submerged ship. Aw, on the other hand, is the so-called wetted cross-sectional area of the waterway excluding the submerged cross-sectional area of the ship. Finally, S1 is defined to be As/ Ac . For rectangular cross-sectional areas, As $= B \cdot T$ and $Ac = w \cdot h$ where w refers to the width of the waterway.

We are considering open water in a lateral, i.e. horizontal sense (no breadth restrictions), and according to Barrass, the effective width, w, for open water should be taken to range from about w 8·B for oil tankers to about w 10.5·B for passenger vessels. Also, for the Barrass formula to work, the assumed ratio of water depth to draught should lie between $1.1 < h/T < 1.5$. Finally the speed in equation (7) is measured in knots whereas in equation (6) it is measured in m/s. We convert the speed in equation (7) to m/s and obtain:

$$S \quad (CB/ 30) \cdot \{(B\cdot T/(w\cdot h))/[1 - (B\cdot T/(w\cdot h))]\}2/3 \cdot (V/.514)2.08 \tag{9}$$

$$(CB/ 30) \cdot \{(T/h)/[w/B - T/h]\}2/3 \cdot (V/.514)2.08$$

$$.133 \cdot CB \cdot \{(T/h)/[10 - T/h]\}2/3 \cdot V2.08$$

In the last line we have used the approximation w/B 10 as a representative value for an open waterway as Barrass advocates. Equation (9) is qualitatively and quantitatively very similar to equation (6) in the range $1.1 < h/T < 1.5$. Both are proportional to CB, both are essentially proportional to T/h, and both have what is essentially a velocity-squared dependency. To show that the results do indeed match in the ranges considered, two numerical examples should suffice. If we assume that V = 6.17 m/s = 12 knots, and if h/T = 1.1, then equation (6) gives (1.21· CB) as a calculated value for the amount of squat in meters. For the same parameters, equation (9) renders (1.26· CB) as a calculated result. We see that both equations give essentially the same value! At the other extreme where h/T = 1.5, using the same speed as before, equation (6) gives (.888· CB) whereas equation (9) gives (1.008· CB) in meters. Again, the results are close given the uncertainties in estimating w/B.

Barrass' formula is a popular one because hand-held calculators can be used to determine the amount of squat. And the amount calculated is not dependent on the specifics of the ship, i.e., no physical dimensions are needed... other than the general form of the vessel which is captured by the value of the block coefficient, CB. In fact, Barrass goes a step further and gives even more simplified expressions for squat based on his underlying equation, equation (7), to make it even easier for the pilot to make estimates. As far as we know, this is the first time equation (7) has been derived from a more complete theory. Remember that Barrass' formula was based solely on empirical observations. Equation (6), on the other hand, is based on theoretical work, dating back to Tuck.

3. SCHMIECHENS' FORMULA DERIVED IN A DIFFERENT LIMIT

Again, we start out with equation (3). However, at the outset of turbulence which would indicate approaching a phase transition, we claim that Fnh must approach 1. In fact, Fnh will never reach one, for otherwise, the vessel would lift up out of the water. A good representative value for when significant, measurable squat sets in, is the limit where Fnh approaches .7. Remember that sizable squat can never occur below Fnh .3. It stands to reason that a larger value for Fnh is needed because a larger value indicates more speed relative to depth, and more speed indicates more water being flushed under the keel. Due to the Bernoulli effect this is precisely what pulls the ship down.

Another way to view this is from energy considerations. As a vessel assumes enough speed to generate significant squat, it comes one step closer to overcoming its captive wave. An exponential increase in hp is necessary to build up that speed, however, because part of that hp is now being used up to pull the vessel down. This happens to aircraft as well; as an aircraft approaches Mach 1 a sizable proportion of the power is now being expended in attempting to break free of its captive wave and not just increase its speed another notch. When approaching Mach 1, an aircraft experiences turbulence due to its stronger interaction with its resisting captive wave. The increased hp necessary in a ship is due to the increased resistance from the captive wave as the speed increases, and this increase in resistance (drag force) is not linear in V. Power in crude terms is Force · Velocity where the velocity is in the forward direction, and the force is the retarding force acting against this motion. (We are looking at the work done per unit time by an external agent, the engines of the ship.) Since the retarding force for a solid object moving through a fluid at significant speed is proportional to V2 (Raleigh equation (Serway & Jewett, 2004; Tipler, 2004)), the power expended in driving a vessel forward must be proportional to V3. Part of this power is what produces squat and we therefore expect a V3 dependency for squat as well at speeds of interest.

In the limit where Fnh approaches .7, the following approximation formula works [10]:

$$Fnh2/\sqrt{(1- Fnh2)} \quad 2 \cdot Fnh2 \qquad (10)$$

Then equation (3) reduces to

$$S/T = C3 \cdot Fnh3 \qquad (11)$$

where C3, defined as $C3 = 4.8 \cdot CB \cdot B/ Lpp'$, is a new constant which depends only on the characteristics of the ship. Equation (10) was first obtained by Schmiechen. We note that in equation (11), S/T is proportional to V3, as expected, versus the V2 obtained previously in equation (5). We maintain that equation (11) is a better simplified formula to calculate and approximate ship squat, versus equations (5) or (6), because of the above mentioned physical considerations. Experimental verification of a V3 (versus V2) dependency has been obtained (Akudinov & Jakobsen, 1995) with a model of the Herald of Free Enterprise.

For numerical estimates, we can again assume that Lpp'/B 7. Then, numerically, the relative squat can be determined from equation (11) to be

$$S/T \quad 4.8 \cdot CB /7 \cdot V3/(g \cdot h)3/2 \qquad (12)$$

$$CB /45 \cdot (V/\sqrt{h})3$$

This is a simple formula to remember and to work with. For V = 6.17 m/s = 12 knots and h = 10m, one obtains for S/T a value of .16 CB. However, if V = 6.17 m/s and h = 5m, then the relative amount of squat is S/T = .468 CB, an almost three-fold increase, even for vessels which are very finely formed indicated by low values for CB. Equation (12) does not suffer from the restriction that $1.1 < h/T < 1.5$ as equation (7) does. Nor is the questionable approximation of w $10 \cdot B$ for open waterways invoked. If we are dealing with an open waterway in the sense that there is no breath restriction, then the proper limit to take, mathematically, is w .

4. CONCLUSION AND REMARKS:

We claim that Barrass' simplified formula, or any formula indicating a V2 dependency for ship squat, cannot be correct at speeds where squat is significant. It is obtained in an incorrect limit, and hence, cannot be valid. Ship squat is a hydrodynamic effect relating to a ship overcoming its captive wave and approaching a phase transition. Hence the Froude depth number should be approaching unity and not zero. Whenever turbulence sets in, it is a sign that the retarding force is proportional to V2 versus V.

Hence the power expended is proportional to V3. Since part of the power goes into generating squat, S/T should also be proportional to V3. S/T being proportional to V2 holds only for relatively low speeds and steady state flow (Stokes equation (Serway & Jewett, 2004; Tipler, 2004)), not indicated experimentally by the need for exponentially increased hp or other conditions.

In fact, in several of Barrass' papers (Barrass, 1979a; Barrass, 1979b), it is recognized that the onset of ship squat is accompanied by the following quoted tell-tale signs:

1) Wave-making increases at the for'd end of the ship
2) The ship becomes more sluggish to maneuver
3) The r.p.m. indicator will show a decrease. If the ship is in open water, i.e. without breadth restrictions, this decrease may be 15% of the service number of revolutions. If a ship is in a confined channel, this decrease in r.p.m. can be about 20% of the normal value.
4) There will be a drop in speed. If the ship is in open water the speed reduction may amount to about 30%. If the ship is in a confined channel, the drop may amount to 60% of service speed.
5) The ship may start to vibrate suddenly because of the entrained water effect, causing the natural hull frequency to become resonant with another frequency.

These are all experimental manifestations of the onset of turbulent conditions, when an object attempts to break free of its captive wave, and not low speeds and steady-state flows.

Schmiechen has correctly identified the correct limit, and come up with the correct approximation. Hence we advocate the use of equations (11) or (12) as good benchmarks to mariners for estimating squat. Note that equation (12), in particular, can be used with relative ease using a simple hand-held calculator. The specific dimensions of the ship do not come into play... only the block coefficient which is determined by the general type of vessel.

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